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## MUON DECAY ASYMMETRIES FROM $K_L^0 \rightarrow \pi^0 \mu^+ \mu^-$ DECAYS

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### ABSTRACT

We have examined the decay  $K_L^0 \rightarrow \pi^0 \mu^+ \mu^-$  in which the branching ratio, the muon energy asymmetry and the muon decay asymmetry could be measured. In particular, we find that within the Standard Model the longitudinal polarization ( $P_L$ ) of the muon is proportional to the direct CP violating amplitude. On the other hand the energy asymmetry and the out-of-plane polarization ( $P_N$ ) depend on both indirect and direct CP violating amplitudes. Although the branching ratio is small and difficult to measure because of background, the asymmetries could be large  $\mathcal{O}(1)$  in the Standard Model. A combined analysis of the energy asymmetry,  $P_L$  and  $P_N$  could be used to separate indirect CPV, direct CPV, and CP conserving contributions to the decay.

### 1 Introduction

There are three possible contributions to the  $K_L^0 \rightarrow \pi^0 l^+ l^-$  decay ampli-

tude: 1) direct CP-violating contribution from electroweak penguin and W-box diagrams, 2) indirect CP-violating amplitude from the  $K_L \rightarrow \pi^0 l^+ l^-$  component in  $K_L$ , and 3) CP-conserving amplitude from the  $\pi^0 \gamma \gamma$  intermediate state 1, 2). The sizes of the three contributions depend on whether the final-state lepton is an electron or a muon. The CP-conserving two-photon contribution to the electron mode is expected to be  $(1 - 4) \times 10^{-12}$ , based on  $K_L \rightarrow \pi^0 \gamma \gamma$  data. Although suppressed in phase space, this contribution to the muon mode is comparable to the electron mode because of the scalar form factor which is proportional to lepton mass<sup>3)</sup>. The interesting direct CP-violating component must be extracted from any signal found for  $K_L \rightarrow \pi^0 l^+ l^-$  in the presence of two formidable obstacles: the theoretical uncertainty on contamination from indirect CP-violating and CP-conserving contributions, and the experimental background from  $l^+ l^- \gamma \gamma$ <sup>4)</sup>.

To subtract the indirect CP and CP conserving contributions, several authors have examined the use of measurements from  $K_L \rightarrow \pi^0 \gamma \gamma$ ,  $K_S \rightarrow \pi^0 e^+ e^-$ , as well as the lepton energy asymmetry<sup>2, 5, 6, 7, 8)</sup>. With better measurements expected in the near future, it is useful to reexamine  $K_L^0 \rightarrow \pi^0 l^+ l^-$  decays<sup>9, 10)</sup>.

In this paper, we examine if the muon decay asymmetries give additional constraints on CP violation in  $K_L^0 \rightarrow \pi^0 \mu^+ \mu^-$ . Indeed, we find that within the Standard Model the P-odd longitudinal polarization of the muon is non-zero only if the direct CP violating amplitudes are non-zero. We also find that other asymmetries that involve the polarization of both muons can be constructed to isolate the direct and indirect contributions to CP violation. We will proceed in analogy to the charged version of the decay,  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  which has been analyzed extensively<sup>11)–16)</sup>. The structure of  $K_L \rightarrow \pi^0 l^+ l^-$  decays is more complex than that of  $K^+ \rightarrow \pi^+ l^+ l^-$  decays because of CP suppression. In particular, the one-photon intermediate state contribution to the vector form factor ( $F_V$ ), which is expected to be almost real and dominant for  $K^+ \rightarrow \pi^+ l^+ l^-$  decays, is CP-suppressed in  $K_L^0 \rightarrow \pi^0 l^+ l^-$  decays.

The  $K_L^0 \rightarrow \pi^0 l^+ l^-$  decay process can have contributions from scalar, vector, pseudo-scalar and axial-vector interactions, with corresponding form factors,  $F_S$ ,  $F_V$ ,  $F_P$ , and  $F_A$ <sup>11)</sup>:

$$\begin{aligned} \mathcal{M} = & F_S \bar{u}(p_l, s) v(\bar{p}_l, \bar{s}) + F_P \bar{u}(p_l, s) \gamma_5 v(\bar{p}_l, \bar{s}) \\ & + F_V p_k^\mu \bar{u}(p_l, s) \gamma_\mu v(\bar{p}_l, \bar{s}) + F_A p_k^\mu \bar{u}(p_l, s) \gamma_\mu \gamma_5 v(\bar{p}_l, \bar{s}) \end{aligned} \quad (1)$$

Here  $p_k$ ,  $p_\pi$ ,  $p_l$ , and  $\bar{p}_l$  are the kaon, pion, lepton, and antilepton 4-momenta. Transverse muon polarization effects arise from interference between terms with non-zero phase differences, while longitudinal polarization results from interference between terms with the same phase.

The scalar form factor,  $F_S$ , is expected to get a contribution from only the two-photon intermediate state,  $K_L \rightarrow \pi^0 \gamma \gamma \rightarrow \pi^0 \mu^+ \mu^-$ . Pseudo-scalar,  $F_P$ , and axial-vector,  $F_A$ , get contributions from the short-distance “Z-penguin” and “W-box” diagrams only, where the dominant term arises from t-quark exchange; both form factors therefore depend on  $V_{ts} V_{td}^*$ , with a small contribution from the charm quark.  $F_V = F_V^+ + F_V^-$  where  $F_V^+$  is the CP-even contribution from the two photon process and is proportional to  $p_k \cdot (p_l - \bar{p}_l)$ .  $F_V^-$  is the CP-odd contribution, the sum of  $F_V^{MM}$ , the indirect CP violation in  $K_L$  decays, and  $F_V^{dir}$ , the short distance, direct CP violating contribution. Unlike  $K^+$ , all the amplitudes in the  $K_L$  decay are likely to be of the same order of magnitude, and polarization effects should therefore be large ( $\mathcal{O}(1)$ ), unless there are strong cancellations<sup>17</sup>.

The symmetries of the decay are clearest in the  $l^- l^+$  CM frame. In this reference frame,  $p_l = (E, \vec{p})$ ,  $\bar{p}_l = (E, -\vec{p})$  and  $\lambda, \mu$  are the helicities of the lepton and anti-lepton, respectively. Using this notation, the helicity amplitudes,  $M_{\mu\lambda}(\hat{p})$ , in terms of the four form factors  $F_S, F_P, F_V, F_A$  are:

$$\begin{aligned}
M_{++}(\hat{p}) &= -F_S \frac{p}{m} + F_P \frac{E}{m} - F_V p_k \cos \theta + F_A E_k \\
M_{--}(\hat{p}) &= +F_S \frac{p}{m} + F_P \frac{E}{m} + F_V p_k \cos \theta + F_A E_k \\
M_{+-}(\hat{p}) &= +F_V \frac{E}{m} p_k \sin \theta - F_A \frac{p}{m} p_k \sin \theta \\
M_{-+}(\hat{p}) &= +F_V \frac{E}{m} p_k \sin \theta + F_A \frac{p}{m} p_k \sin \theta
\end{aligned} \tag{2}$$

Where  $p, E, m$  are the momentum, energy and the mass of the negative lepton, and  $\theta$  is the angle between the negative lepton and the kaon in the  $l^- l^+$  CM frame.  $M_{\mu,\lambda}(\hat{p})$  can depend on various invariants like  $p_k \cdot (p_l + \bar{p}_l)$  but we single out the unit vector  $\hat{p} = \vec{p}/|\vec{p}|$  because it changes under the  $CP$ -operation. Under  $CP$   $M_{\mu,\lambda}(\hat{p}) \rightarrow -M_{-\lambda,-\mu}(-\hat{p})$ . As we have already discussed,  $F_V = F_V^+ + F_V^-$  and  $F_V^\pm(\hat{p}) = \mp F_V^\pm(-\hat{p})$ . In this set of graphs the form factors have no  $\theta$  dependence other than an explicit factor of  $\cos \theta$  in  $F_V^+$ .

These helicity amplitudes can be used to create interesting CP-odd ob-

servables such as the following three:

$$\begin{aligned}
|M_{++}(\hat{p})|^2 - |M_{--}(-\hat{p})|^2 &= -4\text{Re}\{(F_S \frac{p}{m} + F_V^+ p_k \cos \theta)^* \\
&\quad \times (F_P \frac{E}{m} + F_A E_k - F_V^- p_k \cos \theta)\} \\
|M_{+-}(\hat{p})|^2 - |M_{+-}(-\hat{p})|^2 &= +4\text{Re}\{(F_V^+ \frac{E}{m})^* \\
&\quad \times (F_V^- \frac{E}{m} - F_A \frac{p}{m})\} p_k^2 \sin^2 \theta \\
|M_{-+}(\hat{p})|^2 - |M_{-+}(-\hat{p})|^2 &= 4\text{Re}\{(F_V^+ \frac{E}{m})^* \\
&\quad \times (F_V^- \frac{E}{m} + F_A \frac{p}{m})\} p_k^2 \sin^2 \theta
\end{aligned} \tag{3}$$

The difference between the last two asymmetries isolates the interference between the  $CP = +1$  part of  $F_V$  and purely direct CP violating amplitudes.

The above asymmetries involve the measurement of the polarization of both leptons in the same event. It is possible to get direct information about CP violating amplitudes from measuring asymmetries for one of the leptons only, even though the asymmetry is not intrinsically CP-violating. It is well-known<sup>18)</sup> that the out-of-plane polarization of the muon, transverse to the plane formed by the  $\pi$  and the lepton momenta, gets contributions from CP-violating amplitudes, but several effects are mixed up and it is not possible to give a separation of the direct and the indirect pieces. It is, however, not well-known that more information can be obtained from the parity-violating single lepton longitudinal asymmetries, even though they are not intrinsically CP-violating. The longitudinal polarizations are given in the lepton-lepton c.m. frame as:

$$\begin{aligned}
P'_L(\hat{p}) &= [|M_{++}|^2 + |M_{-+}|^2 - |M_{+-}|^2 - |M_{--}|^2] / \rho \\
&= [-4\text{Re}\{(F_S \frac{p}{m} + F_V p_k \cos \theta)^* (F_P \frac{E}{m} + F_A E_k)\} \\
&\quad + 4 \frac{pE}{m^2} p_k^2 \sin^2 \theta \text{Re}(F_A^* F_V)] / \rho, \\
\bar{P}'_L(-\hat{p}) &= [|M_{++}|^2 + |M_{+-}|^2 - |M_{-+}|^2 - |M_{--}|^2] / \rho \\
&= [-4\text{Re}\{(F_S \frac{p}{m} + F_V p_k \cos \theta)^* (F_P \frac{E}{m} + F_A E_k)\} \\
&\quad - 4 \frac{pE}{m^2} p_k^2 \sin^2 \theta \text{Re}(F_A^* F_V)] / \rho,
\end{aligned}$$

$$\rho = |M_{++}|^2 + |M_{-+}|^2 + |M_{+-}|^2 + |M_{--}|^2. \quad (4)$$

We use the notation  $P'_L$  here to distinguish the polarization components measured in the  $\bar{l}l$  cm to distinguish them from the components  $P_L$  measured in the kaon rest frame, to be used later.

These polarizations have the property that although they are not CP violating, they are zero if there is no direct CP violation. The combination  $(P'_L(\hat{p}) + P'_L(-\hat{p}) + \bar{P}'_L(\hat{p}) + \bar{P}'_L(-\hat{p}))$  cancels the interference between pairs of CP violating amplitudes and leaves a pure CP-violating observable. Similarly  $(P'_L(\hat{p}) + P'_L(-\hat{p}) - \bar{P}'_L(\hat{p}) - \bar{P}'_L(-\hat{p}))$  is CP-even. The angular dependence can be integrated to give a particularly simple result:

$$\begin{aligned} \langle P'_L + \bar{P}'_L \rangle &= -16 \text{Re}\{ (F_S \frac{p}{m} + \frac{1}{3} F_V^+(\theta=0) p_k)^* (F_P \frac{E}{m} + F_A E_k) \} / \rho \\ \langle P'_L - \bar{P}'_L \rangle &= \frac{32}{3} \frac{pE}{m^2} p_k^2 \text{Re}(F_A^* F_V^-) / \rho \end{aligned} \quad (5)$$

The first of these is C-even and purely direct, and the second is C-odd and contains both direct and indirect amplitudes. It should be noted that for these asymmetries the muon and anti-muon polarizations can be measured separately over the same part of phase space. Indeed, if a complete angular analysis of one lepton's polarization can be performed, four out of the 6 independent products the form factors can be determined.

We will show our numerical results in the kaon rest frame. We concentrate on four measurable quantities: the total decay rate, the energy asymmetry between the muons and the out-of-plane and the longitudinal components of the muon polarization. The in-plane transverse polarization should also be considered when designing an experiment.

For the purposes of discussing possible experiments, it is useful to have the lepton polarization given in a covariant way. The form is the same as for  $K^+ \rightarrow \pi^+ l^- l^+ 11$ ):

$$\begin{aligned} P(s) &= \{-2 \text{Re}(F_S F_P^*) m(s \cdot \bar{p}_l) \\ &+ 2 \text{Re}(F_V F_A^*) m[2(s \cdot p_k)(\bar{p}_l \cdot p_k) - m_K^2(s \cdot \bar{p}_l)] \\ &- 2 \text{Re}(F_P F_V^*) [-\frac{1}{2} q^2(p_k \cdot s) + (p_k \cdot p_l)(\bar{p}_l \cdot s)] \\ &+ 2 \text{Re}(F_S F_A^*) [\frac{1}{2}(q^2 - 4m^2)(p_k \cdot s) - (\bar{p}_l \cdot s)(p_l \cdot p_k)] \\ &+ [2 \text{Im}(F_P F_A^*) + 2 \text{Im}(F_S F_V^*)] \epsilon^{\mu\nu\rho\sigma} p_{k\mu} p_{l\nu} \bar{p}_{l\rho} s_\sigma \} / (m^2 \rho), \end{aligned} \quad (6)$$

where  $q^2 = (p_l + \bar{p}_l)^2$  and  $s$  denotes the covariant spin vector of the lepton. The decay rate in the kaon rest frame is given as <sup>11)</sup>:

$$\begin{aligned}
m^2 \rho_0(E_l, \bar{E}_l) &= |F_S|^2 \frac{1}{2}(q^2 - 4m^2) + |F_P|^2 \frac{1}{2}q^2 \\
&+ |F_V|^2 m_k^2 (2E_l \bar{E}_l - \frac{1}{2}q^2) \\
&+ |F_A|^2 m_k^2 [2E_l \bar{E}_l - \frac{1}{2}(q^2 - 4m^2)] \\
&+ 2\text{Re}(F_S F_V^*) m m_k (\bar{E}_l - E_l) \\
&- 2\text{Re}(F_P F_A^*) \frac{m}{2} (m_k^2 - M_\pi^2 + q^2). \tag{7}
\end{aligned}$$

The total rate is given by

$$\Gamma = \frac{m^2}{2m_k} \int \rho_0(E_l, \bar{E}_l) \frac{dE_l d\bar{E}_l}{(2\pi)^3}. \tag{8}$$

The spin vector  $s_L$  in the direction of the  $\mu^-$  momentum in the kaon rest frame is

$$s_L = (p, E_l \sin \theta_{l\bar{l}}, 0, E_l \cos \theta_{l\bar{l}})/m \tag{9}$$

where we take the decay to be in the  $x - z$  plane with  $\vec{p}_l$  pointing in the  $z$ -direction. Then the longitudinal polarization of  $\mu^-$  in the kaon rest frame (not the same as Eq.4 which is in the  $\mu^- \mu^+$  c.m.) is given as

$$\begin{aligned}
P_L &= [-2\text{Re}(F_S F_P^*) (\bar{E}_l - \frac{E_l}{p_l^2} \vec{p}_l \cdot \vec{p}_l)) \\
&+ 2\text{Re}(F_V F_A^*) m_k^2 (\bar{E}_l + \frac{E_l}{p_l^2} \vec{p}_l \cdot \vec{p}_l)) \\
&+ 2\text{Re}(F_P F_V^*) m_k (m + \frac{m}{p_l^2} \vec{p}_l \cdot \vec{p}_l)) \\
&+ 2\text{Re}(F_S F_A^*) m_k (-m + \frac{m}{p_l^2} \vec{p}_l \cdot \vec{p}_l))] p_l / (m^2 \rho_0) \tag{10}
\end{aligned}$$

where  $\vec{p}_l(\vec{p}_l)$  and  $E_l(\bar{E}_l)$  are the momentum and energy of  $\mu^-(\mu^+)$ . Notice that in this frame  $F_V^+$  is odd under  $E_l \leftrightarrow E_{\bar{l}}$ , while  $F_V^-$  and the other form factors are even under the same interchange.

The transverse (out of decay plane) polarization perpendicular to the muon momentum vector in the kaon center of mass frame is given as

$$P_N = -2[\text{Im}(F_S F_V^*) + \text{Im}(F_P F_A^*)] p_l \bar{p}_l \sin \theta_{l\bar{l}} / (m^2 \rho_0). \tag{11}$$

It should be noted that  $F_P$  and  $F_A$  are in phase and therefore do not contribute to  $P_N$ .

For completeness, we also give the expression for the transverse, in-the-plane polarization  $P_T$ . For this the spin vector is  $s_T = (0, \cos \theta_{l\bar{l}}, 0, -\sin \theta_{l\bar{l}})$ , and the expression is as follows:

$$\begin{aligned}
P_T &= -[2\text{Re}(F_S F_P^*) m \\
&+ 2\text{Re}(F_P F_V^*) m_K E_l \\
&+ 2\text{Re}(F_S F_A^*) m_K E_l \\
&+ 2\text{Re}(F_V F_A^*) m m_K^2] \bar{p}_l \sin \theta_{l\bar{l}} / (m^2 \rho_0).
\end{aligned} \tag{12}$$

It depends on the same quantities as  $P_L$  and, depending on the experimental configuration, the measured quantity may be a linear combination of the two. ( $P'_L$  is the linear combination of  $P_L$  and  $P_T$  given by the rotation of the lepton spin through the angle between the kaon and the lepton momenta as seen in the rest frame of the lepton.)

We now consider values of the form-factors that we will use in estimating the polarization.

$F_V^{MM}$ : The decays  $K_S \rightarrow \pi^0 l^+ l^-$  have been studied in chiral perturbation theory extensively. The same framework of analysis is often applied to the similar decay of  $K^+ \rightarrow \pi^+ l^+ l^-$ . In D'Ambrosio *et al*<sup>(6)</sup>, the decays are analyzed beyond the leading order  $\mathcal{O}(p^4)$ . The vector form factor for  $K_S$  decays is parametrized as

$$F_V^S = -G_F \frac{\alpha}{4\pi} (a_S + b_S z) - \frac{\alpha}{4\pi m_k^2} W^{\pi\pi}(z) \tag{13}$$

The function  $W^{\pi\pi}$  comes from the pion loop contribution, which is estimated to  $\mathcal{O}(p^6)$  using  $K \rightarrow \pi\pi\pi$  data, and the rest of the contributions are parametrized in the linear term. Using ideas from VMD a further assumption is sometimes made,

$$b_S = \frac{a_S}{M_V^2/m_k^2} = \frac{a_S}{2.5} \tag{14}$$

where  $M_V$  is the vector meson mass. The vector form factor for indirect CP violation in  $K_L \rightarrow \pi^0 \mu^+ \mu^-$  is then

$$F_V^{MM} = \epsilon F_V^S \tag{15}$$

The value of  $a_S$  is quite unknown, however it is considered to be  $\mathcal{O}(1)$ . The study of the similar decay  $K^+ \rightarrow \pi^+ l^+ l^-$  gives a value of the equivalent parameter to be  $-0.59 \pm 0.01$ ,<sup>19)–21)</sup> therefore we use the value,  $-0.6$ , for our numerical results.

Most of the other values are taken from Donoghue and Gabbiani<sup>5)</sup>.  $F_V^{dir}$ ,  $F_A$ , and  $F_P$ : These form factors get contributions from short distance box and penguin diagrams. We use the notation from Donoghue and Gabbiani<sup>5)</sup>:

$$\begin{aligned} F_V^{dir} &= 2 \frac{G_F}{\sqrt{2}} i y_{7V} \text{Im} \lambda_t \\ F_A &= -2 \frac{G_F}{\sqrt{2}} i y_{7A} \text{Im} \lambda_t \\ y_{7V} &= 0.743 \alpha, \text{ at } m_t = 175 \text{ GeV} \\ y_{7A} &= -0.736 \alpha, \text{ at } m_t = 175 \text{ GeV} \\ \lambda_t &= V_{td} V_{ts}^*. \end{aligned} \quad (16)$$

We have used  $\text{Im} \lambda_t = 10^{-4}$  in our numerical calculation. Similar to the case of  $K^+$  decay  $F_P$  is related to  $F_A$  by<sup>11)</sup>

$$F_P = -m_l \left(1 - \frac{f_-}{f_+}\right) F_A \quad (17)$$

where  $f_+$  and  $f_-$  are charged current semileptonic decay form factors of  $K_L$ .  $F_S$ : Since<sup>5)</sup> is concerned only with electron final states, the scalar contribution is negligible and they do not calculate it. Therefore we use the earlier result of Ecker *et al*<sup>18)</sup>. They have calculated  $F_S$  to  $\mathcal{O}(p^4)$  as

$$\begin{aligned} F_S &= \frac{i G_8 \alpha^2}{4\pi} m E(z) \\ E(z) &= \frac{1}{\beta z} \log\left(\frac{1-\beta}{1+\beta}\right) [(z - r_\pi^2) F(z/r_\pi^2) - (z - 1 - r_\pi^2) F(z)]. \end{aligned} \quad (18)$$

$z = (p_l + \bar{p}_l)^2 / m_k^2$ ,  $r_\pi = m_\pi / m_k$ , and  $\beta = \sqrt{1 - 4 \frac{m_l^2}{m_k^2} z}$ .  $F(z)$  is a known function described by Ecker *et al*.

$F_V^+$ : For this we return to<sup>5)</sup>. Using chiral perturbation theory they obtain

$$F_V^+ = 2 G_8 \alpha^2 p_k \cdot (p_l - \bar{p}_l) K(s) \quad (19)$$

$$K(s) = \frac{B(x)}{16\pi^2 m_k^2} \left[ \frac{2}{3} \ln\left(\frac{M_\rho^2}{-s}\right) - \frac{1}{4} \ln\left(\frac{-s}{m^2}\right) + \frac{7}{18} \right] \quad (20)$$



where  $s = (p_l + \bar{p}_l)^2$ ,  $x = s/4M_\pi^2$ , and  $B(x)$  is a complex function described in 5).  $G_8$  denotes the octet coupling constant in chiral perturbation theory. We use the value  $G_8 = \frac{G_F}{\sqrt{2}}|V_{ud}V_{us}^*|g_8$  with  $g_8^{tree} = 5.1$  5). The value of  $g_8^{loop} = 4.3$  from a higher order calculation does not alter our conclusions significantly 22). The function contains a free parameter,  $\alpha_V = -0.72 \pm 0.08$ , determined from experimental  $K_L \rightarrow \pi^0 \gamma \gamma$  data 9). We have used  $\alpha_V = -0.70$  for our numerical results. It should be noted that Heiliger *et al* 3) have performed an analysis of both  $F_S$  and  $F_V^+$  in a two component (pion loop and vector meson dominance (VMD)) model. This calculation, however, needs to be checked against the latest data on  $K_L \rightarrow \pi^0 \gamma \gamma$ . Recently, Gabbiani and Valencia have pointed out that a more complete formulation of  $F_V^+$  requires three free parameters which can be obtained from  $K_L \rightarrow \pi^0 \gamma \gamma$  data 8).

Using the above discussed values for the various form factors the total decay rate is about  $\sim 1.1 \times 10^{-11}$ , and it is dominated by the scalar interaction which makes most of the contribution for mu-mu mass above 280 MeV. This is because of the two pion loop contribution embodied in the form factor  $F_S$ . Unlike  $F_S$ , the rest of the form factors are expected to give contributions that fall with  $q^2$ . The contribution from  $F_P$  is small because of the suppression due to the lepton mass. The region  $\sqrt{q^2} < 280$  MeV will be affected by interference effects as shown in figure 1. It is interesting to note that the destructive interference causes the decay rate to be almost zero in the region where the  $\mu^+$  is at rest in the kaon rest frame. The longitudinal and out-of-plane polarizations of the  $\mu^-$  are shown in figure 2. The energy asymmetry and the out-of-plane polarizations are large, but they also have large dependence on the parameter  $a_S$ . If the out-of-plane polarization is used to constrain  $a_S$  then the short distance physics could be extracted by fitting the measured distribution for the longitudinal polarization which mainly comes from  $Re(F_S F_A^*)$  and  $Re(F_A F_V^*)$ .

The modes  $K_L \rightarrow \pi^0 e^+ e^-$  and  $K_L \rightarrow \pi^0 \mu^+ \mu^-$  have not as yet been observed; the current best limits on the branching ratios for  $K_L \rightarrow \pi^0 l^+ l^-$  were obtained by the KTeV experiment at FNAL;  $B(K_L \rightarrow \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10}$  and  $B(K_L \rightarrow \pi^0 e^+ e^-) < 5.1 \times 10^{-10}$  23, 24). These limits were based on 2 observed events in each case, and expected backgrounds of  $0.87 \pm 0.15$  for the muon mode and  $1.06 \pm 0.41$  for the electron mode. The main backgrounds for the muon mode were estimated to be from  $\mu^+ \mu^- \gamma \gamma$  ( $0.37 \pm 0.03$ ) and  $\pi^+ \pi^- \pi^0$  ( $0.25 \pm 0.09$ ), in which both charged pions decay in flight. Of these, the for-

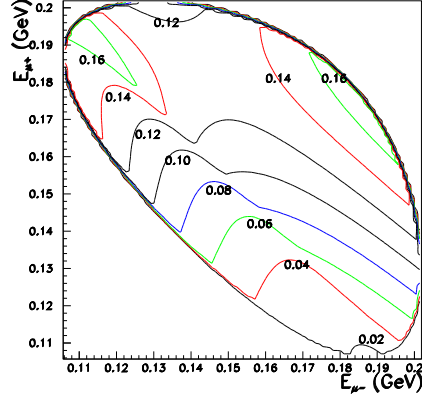


Figure 1: Dalitz decay distribution for  $K_L \rightarrow \pi^0 \mu^+ \mu^-$ . The units for the decay rate contours are arbitrary. The total calculated branching fraction was  $1.1 \times 10^{-11}$ . The form factors and the values of the parameters used are described in the text.

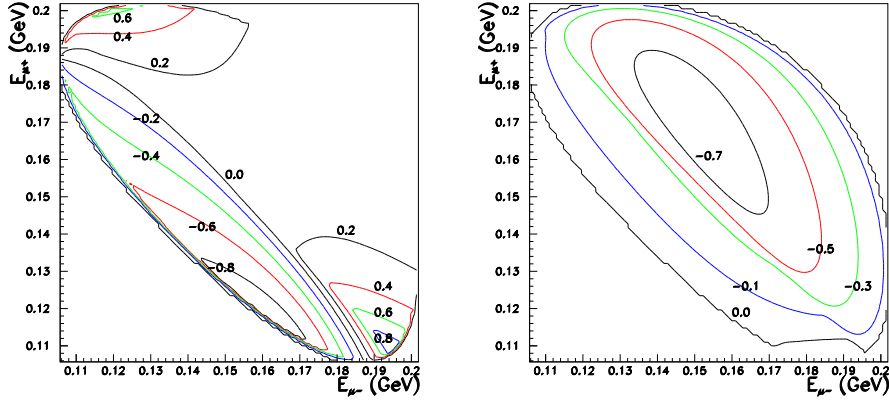


Figure 2: Longitudinal (left) and out-of-plane (right) polarizations of  $\mu^-$  in  $K_L \rightarrow \pi^0 \mu^+ \mu^-$  decay plotted as a function of  $\mu^+$  and  $\mu^-$  energy in the kaon rest frame.

mer background could be irreducible and therefore of great concern. For any future experiment it seems unlikely that the background due to  $\mu^+\mu^-\gamma\gamma$  can be lowered. The signal to background ratio, assuming that only  $\mu^+\mu^-\gamma\gamma$  will contribute in a future experiment, will be around 1/5, if the standard-model signal is taken as  $B(K_L \rightarrow \pi^0\mu^+\mu^-) \sim 5 \times 10^{-12}$ .<sup>17)</sup>

Measuring the muon polarization asymmetries in  $K_L \rightarrow \pi^0\mu^+\mu^-$ , together with the branching ratio and the lepton energy asymmetry, could be a good way of defeating the intrinsic background from CP-conserving and indirect CP-violating amplitudes and the experimental background from  $\mu^+\mu^-\gamma\gamma$ . The large predicted asymmetries should be easy to measure with sufficient statistics at new intense proton accelerators such as the Brookhaven National Laboratory AGS, the Fermilab main injector, or the Japanese Hadron Factory. An examination of the functional form of the form factors  $F_S$  and  $F_V$  is needed to see if the present function form in terms of the parameters  $a_V$  and  $a_S$  is adequate. Examination of how to measure the different components of the polarization in the laboratory as well as the  $\mu\mu\gamma\gamma$  background is needed to understand the dilution of the asymmetries on the Dalitz plot.

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